AN APPLICATION OF AN ECONOMETRIC MODEL TO DESCRIBE THE BEHAVIOR OF BUYERS AND SELLERS

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## NAVAL POSTGRADUATE SCHOOL Monterey, California



## THESIS

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to Describe the Behavior of Buyers and Sellers

by<br>James Fredrick Callahan

June 1974

Thesis Advisor:
C. A. Peterson

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and explicit treatment of the time variable were discussed.

An Application of an Econometric Model to Describe the Behavior of Buyers and Sellers

## by

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## I. INTRODUCTION

The problem of determining the appropriate price for residential real estate is one which confronts both the buyer and the seller. Presently, it appears that there are several hueristic methods used to assist in the solution of this problem. An example of such a hueristic is that a house should be priced at or bought for a certain number of dollars per square foot. This hueristic might be modified to include consideration of such things as lot size, school district, etc. While these methods may be adequate, the question of consistency of the results may be raised.

The area of concern of this paper was to examine whether or not there exist discernable pricing models in the residential real estate market. While the characteristics of a market of this type vary from area to area as well as with time, there are some general similarities. In communities which are in a growth phase of development or are in a stable phase with positive expectations, the seller can be considered as a price setter and the buyer as a price taker. If a community is in a reduction phase of development or is stable with negative expectations, the role of buyer and seller are reversed. However, since the market discussed is imperfect in an economic sense, these roles are subject to modification and compromise with respect to individual transactions.

The price tendered by the seller and the price offered by the buyer are the result of the consideration of many variables.


The market characteristics represent only a portion of the things considered. As a result, the prices proposed by the buyer and the seller often differ initially. This can be attributed to different perceptions as to the variables considered in the relevant set by the buyer and the seller, as well as different levels of importance assigned to the same variables. Thus, the behavior of buyers and sellers in this market presents a very complex system to model.

The general objective of this paper is to develop pricing models for both buyer and seller and to examine the economic behavior of buyer and seller. These models are proposed in a form consistent with the desire to test some economic hypotheses and consistent with the data available. As a result, there is a high level of abstraction associated with the models used. The methodology utilized was to specify the model, estimate the coefficients of the model with the data available by econometric methods and, by means of additional statistical analysis, test relevant hypotheses which allowed an economic interpretation of the statistical results.


## II. DATA BASE

The econometric analysis is, of course, dependent upon the availability of data. The data used in this paper represents one hundred eighty seven residential real estate sales made in Monterey, California from April 1971 to December 1972. The source of this data was the Monterey Multiple Listing Service. As utilization of the Multiple Listing was not mandatory then, the data base does not contain a record of all sales made during this perod.

The variables recorded for each sale are shown in Table I.

## TABLE I

| VARIABLE | DESCRIPTION | UNIT OF MEASURE | MEAN | RANGE | STANDARD DEVIATION |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X (1) | Asked Price | \$ | 38087 | 89500-15750 | 13256 |
| $\mathrm{X}(2)$ | Sale Price | \$ | 36212 | 70000-13750 | 12463 |
| X(3) | House Size | SF | 1516 | 3200-400 | 547 |
| $\mathrm{X}(4)$ | Bedrooms | EA | 2.840 | 6-1 | . 738 |
| $\mathrm{X}(5)$ | Bathrooms | EA | 1.738 | 4-1 | . 625 |
| $\mathrm{X}(6)$ | $\mathrm{X}(1)-\mathrm{X}(2)$ | \$ | 1876 | 19500-0 | 2068 |
| $\mathrm{X}(7)$ | Qtr of Sale | $0 / 1$ | - | - | - |
| X (8) | Qtr of Sale | 0/1 | - | - | - |
| $\mathrm{X}(9)$ | Qtr of Sale | 0/1 | - | - | - |



Data on other relevant variables was not recorded. Some variables in this category were: (1) the size of the lot upon which the house is constructed, (2) the area of the city in which the property is located and (3) the amount of time the property was on the market. Whether or not the data available was adequate to satisfactorily develop the desired models will be discussed in a later section.
III. GENERAL CHARACTERISTICS OF ECONOMETRIC ANALYSIS

The models used in this paper were all developed using stepwise linear least squares regression. They are of the linear form:

$$
Y=X \beta+\varepsilon
$$

where

$$
\begin{array}{ll}
Y=\left[\begin{array}{l}
Y_{1} \\
\cdot \\
\cdot \\
\cdot \\
Y_{1, N}
\end{array}\right] \quad X=\left[\begin{array}{cccccc}
1 & X_{21} & \cdot & \cdot & \cdot & X_{K 1} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & X_{2, N} & \cdot & \cdot & \cdot & X_{K N}
\end{array}\right] \\
B=\left[\begin{array}{l}
B_{0} \\
\cdot \\
\cdot \\
B_{K}
\end{array}\right]
\end{array}
$$

or

$$
Y=A X^{\beta} \varepsilon
$$


which transforms into

$$
\ln Y=\ln A+\beta \ln X+\ln \varepsilon
$$

The logarithmic transformation has the same characteristics as the linear form.

The stepwise regression was used to produce estimates of the coefficients, bringing new variables into the model in the order of the magnitude of their contribution to explaining the behavior of the dependent variable. A hypothesis test was conducted to determine if the coefficients estimated were significantly different from zero. This test was a t-test with number of degrees of freedom determined by the number of coefficients being estimated, and the number of data points.

## A. STATISTICAL ASSUMPTIONS

The statistical properties of the models follow from four assumptions which are made:

## 1. Assumption One

For each transaction, the observed value of the dependent variable is the realization of a random variable. The distribution of this variable, in probabalistic form, describes the set of values which the dependent variable might have taken on for a particular transaction, where only one value is observed from this set. The independent variables are nonrandom variables.
2. Assumption Two

The distribution of this random variable is such that it's conditional expectation, given the values of a set of independent variables, is a linear function of this set of values.
3. Assumption Three

The conditional variance of the dependent variable, given the set of values of the independent variables, is a constant.
4. Assumption Four

There exists a random error term, $\varepsilon$, in each of the linear relationships assumed in 2. above. This random error term has a zero expectation and a variance $\sigma^{2}$. Further, each of these randon error terms is independent and normally distributed.

## B. PARTICULAR PROBLEMS

The models developed examined the behavior of asked price and sale price as functions of several variables. Two problems arise with respect to the data available. The first is the explicit treatment of the time variable and the second is the probable multicollineal relationship between some of the independent variables. The treatment given to these problems is given in the following sections.

1. The Time Problem

The time variable has been treated on a quarterly
basis. This has been done to explore the question of whether or not there exists a seasonal fluctuation in the sale price

of properties. Two subproblems arise when a quarterly system is used.
a. Explicit Treatment of Seasonal Variation

The problem of seasonal variation was met by introducing explicit seasonal variables into the models in which time is consdered as a variable. The approach chosen is that used by Klein, et al in Ref. 3. Their method of accounting for seasonal variation requires the introduction seasonal variables in the model. This provides the advantage of showing the number of degrees of freedom used in accounting for seasonal variation. In general, it may be considered that on degree of freedom is lost for each coefficient estimated. For a typical model, this would take the form:

$$
\begin{gathered}
Y=\beta_{0}+\beta_{I} X_{I}+\beta_{N+1} Q_{1}+\beta_{N+2} Q_{2}+\beta_{N+3} Q_{3} \quad I=1, N \\
Q_{j}=\quad \begin{array}{l}
0 \text { if transaction not in QTR } j \quad j=1,3 \\
1 \text { if transaction in QTR } j
\end{array}
\end{gathered}
$$

Therefore, the seasonal variable follows a recurrent pattern:

| Variable | First <br> Quarter | Second <br> Quarter | Third <br> Quarter | Fourth <br> Quarter |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1}$ | 1 | 0 | 0 | 0 |
| $\mathrm{Q}_{2}$ | 0 | 1 | 0 | 0 |
| $\mathrm{Q}_{3}$ | 0 | 0 | 1 | 0 |

In the fourth quarter of each year the values of $Q_{1}, Q_{2}$, and $Q_{3}$ are all zero. This prevents the system from being overdetermined. As a result of this the variation for the fourth quarter is incorporated into the intercept term. The implication of this method is that the relationship between $Y$ and the $Q_{1}$ 's is additive and linear. This means that the quarterly variation causes only shifts in the intercept term. A more general approach would allow changes in both the slope and the intercept as a function of quarter of sale. This would cause the introduction of second order terms in $Q$ and $X$ and complicate the estimation procedure. The reasonableness of limiting the treatment to the simplest type is supported by the fact that more complex treatments would not have an apparent economic interpretation.

## b. Heteroscedasticity

One of the basic assumptions of the econometric analysis was that the conditional expectation of the dependent variable was constant. In dealing with data concerned with events occuring over time, there is a danger that this assumption may be violated. The hypothesis that this assumption was met was tested against the hypothesis that the variance was changing over time.

The conduct of this test is based on a method reported by Theil in Ref. 5. This method supposes that the random errors $\varepsilon$ be split into two groups. The first group contains the first $N / 2$ errors, the second group contains the remaining $N / 2$ errors. If there are an odd number of observations,

the middle term is omitted. If these random errors were observable, the quotient of the summed squares of each group would be distributed as $F(N / 2, N / 2)$. This result follows from the earlier assumptions. Under the equal variance hypothesis, it would be expected that this quotient would be approximately equal to one. If the equal variance assumption were violated this quotient would depart from the neighborhood of one.

However, it is impossible to observe these random errors. While it is tempting to substitute the least squares residuals for these unobservable errors, it can be shown that a quotient so constructed does not have an $F$ distribution. This is due to the fact that the numerator and denominator are not independent. Goldfeld and Quandt, in Ref. 1 showed that a quotient whose numerator and denominator are independent can be derived by splitting the $N$ observations into two equal sets and computing the least squares residuals separately for each set. Then, if the residuals from each group are squared, summed and the quotient formed, the resulting statistic is distributed $F((N / 2)-K(N / 2)-K) . \quad K$ is the number of coefficients estimated in the model. The results of this test for each model are provided in a later section.
2. Multicollinearity

This condition occurs when a partial or exact
linear relationship exists among the explanatory variables. In this paper, this condition occurs due to the fact that three of the explanatory variables used in the models, house size, bedrooms and bathrooms, are interrelated. When one

explanatory variable takes on values that are nearly equal to some multiple of another explanatory variable, the contribution of each of the variables, individually, is difficult to discern. There are two indications of the magnitude of the multicollinearity. The first of these reveals itself in the correlation between two variables. A high correlation between explanatory variables is an indication of multicollinearity. The second is reflected in the change of the standard error of explanatory variables as additional variables enter the model. If multicollinearity is present between a variable in the model and one which is entering, the standard error of the variable in the model will increase while the numerical value of the coefficient is decreasing. This change illustrates the indeterminancy of the variables as multicollinearity increases.

Theil [Ref. 5] suggests two possible treatments if this problem is present to a degree sufficient to warrant correction. The first involves conditional estimating procedures to obtain smaller variances for the estimated coefficients. The second proposes using linear combinations of the multicollinear variables. Both treatments present difficulty. The first in a computational sense, the second in that economic interpretation of the resulting model is not apparent. Since the multicollinearity present in the models developed in this paper was not great, no correction was made for it. The magnitude of multicollinearity in each model is discussed in a later section.



## IV. THE MODELS

The models developed were based on hypotheses concerning the economic behavior of the participants in the real estate market. They were not merely an attempt to find a set of independent variables to describe the behavior of a dependent variable. There were four basic models developed:

1. Sale Price $=f($ Asked Price)
2. Asked Price $=\mathrm{f}($ Size, Bedrooms, Bathrooms)
3. Sale Price $=f($ Size, Bedrooms, Bathrooms, Time)
4. Asked Price - Sale Price $=\mathrm{f}$ (Asked Price, Time) Each was examined in linear form and log linear form.

This section discusses both the statistical and the economic interpretations of the results obtained. The statistical interpretation examines the significance of the coefficients of the independent variables, the coefficient of determination, the heteroscedasticity problem and the multicollinearity problem. It should be noted the statistic $R^{2}$, the coefficient of determination, serves only as a measure of goodness of fit. This is due to the assumption made concerning the nonrandom nature of the independent variables. This precludes association of the concept of correlation with this statistic. Appendix A provides detailed results of the statistical analysis. The economic interpretation examines any economic inferences which were made as a result of the statistical information available.

## A. SALE PRICE AND ASKED PRICE

This model examined sale price as a function of asked price. The purpose of this model was to determine if there was a tendency for sellers to offer a constant percentage discount from their asked price.

1. Statistical Interpretation

For the linear model, the relationship estimated was

$$
X(2)=0.08+0.93 X(1)
$$

The coefficient of $X(l)$ was significantly different from zero at the .01 level of significance based on a t-test with 185 degrees of freedom. The coefficient of determination was equal to .978 , indicating that a good fit was obtained. With only one explanatory variable, multicollinearity was not a problem. The residuals from this model passed the previously described test against heteroscedasticity at the . 01 level of significance with an $F=1.446$ with $(91,92)$ degrees of freedom. For the exponential model, the relationship estimated was

$$
x(2)=.94 X(1)^{1.007}
$$

The exponent of $X(1)$ was significantly different from zero at the . Ol level of significance. The coefficient of determination was equal to .982 , indicating an excellent fit. Multicollinearity was not a problem. The test against heteroscedasticity was passed at the .01 level of significance.



## 2. Economic Interpretation

The linear model indicates that the seller takes a 7\% discount on that protion of the initially asked price which is above $\$ 800$. The ommission of the effect of the constant would only cause a difference of $\$ 56$ in the estimate of a $\$ 3,500$ discount for a $\$ 500,000$ house. In consideration of this, even though the constant term is statistically significant, the economic conclusion is that the seller generally accepts a $7 \%$ discount of the asked price.

The exponential model indicates that the seller takes a slightly smaller discount than indicated by the linear model. The discount estimated is $6 \%$ applied to the asked price raised to the 1.007 power. Since the standard error of the estimated 1.007 is 0.01 , it is apparent that the $99 \%$ confidence interval estimate of this statistic has a lower bound of less than 1. Considering this and other uncertainties involved, it is difficult to determine that one model is preferable to the other. It is concluded that it is quite likely that the seller will discount the asked price by about $6-7 \%$.
B. ASKED PRICE AND SIZE, BEDROOMS AND BATHROOMS This model examined the asked price as a function of size, number of bedrooms and number of bathrooms. The purpose of this model was to determine if there was a consistent relationship between asked price and the physical parameters of the house.


## 1. Statistical Interpretation

For the linear model, the relationship estimated was

$$
X(1)=0.433+1.271 X(3)+0.834 X(5)
$$

The coefficients of $X(3)$ and $X(5)$ were significantly different from zero at the . 01 level of significance. The coefficient of variation was equal to . 734 , indicating a fair fit is obtained. There was some multicollinearity between $X(3)$ and $\mathrm{X}(5)$. The correlation between these variables was . 74 . Examination of the change in the coefficient of determination going from Step 1 where only $X(3)$ is present to Step 2 where $X(5)$ also enters showed an increase from .663 to .734. Concurrently with this increase in the explanatory capability of the model, there was an increase in the standard error of $X(3)$ of about 30 per cent accompanied by a decrease in the numerical value of the coefficient of about 37 percent. This shows that the price paid for increasing the coefficient of determination is increased uncertainty associated with the estimate of the coefficients of the explanatory variables. The coefficient of $X(4)$ was not significantly different from zero at the . 01 level of significance. Due to this it did not enter the model. The residuals from this model passed the test against heteroscedasticity.

For the exponential model, the relationship estimated was

$$
X(1)=.911 X(3)^{.546} X(5)^{.358}
$$



The exponents of $X(3)$ and $X(5)$ were both significantly different from zero at the .01 level of significance. The coefficient of determination was . 769 indicating a slightly better fit than the linear model. Again there was some multicollinearity between $X(3)$ and $X(5)$. The correlation between these variables was . 72. As in the linear model, $X(3)$ entered at Step 1 with $\mathrm{X}(5)$ being added at Step 2. In the process, the standard error of the exponent of $X(3)$ increased by about $26 \%$ while the numerical magnitude of the exponent decreased by about $33 \%$. The addition of $X(5)$ increased the coefficient of determination from . 700 to . 769. As in the linear model, the improved fit was gained at some price. Also, as with the linear model, $X(4)$ did not enter the model. The residuals passed the test against heteroscedasticity at the . Ol level of significance. A comparison of the statistical attributes indicates that the exponential model is slightly superior.

## 2. Economic Interpretation

Both the linear and exponential models indicated that, of the explanatory variables available, only the size of the house and the number of bathrooms contributed to explaining the behavior of the seller in setting the asked price. As would be expected, the size of the house was the more significant of the two. It explained $66 \%$ of the ariation in the linear model and $70 \%$ of the variation in the exponential model. In both cases, the addition of the number of bathrooms added very little. The linear model showed that $\$ 12.71$ per square foot and $\$ 834$ per bathroom may be used as unit costs

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[^0]in setting the asked price. The constant term of $\$ 4,330$ included the effect of other variables unaccounted for by the model.

The exponents in the exponential model have a particular economic interpretation. They are point estimates of the elasticities of asked price for size and number of bathrooms respectively. In this model the results show that, all other things remaining constant, an increase in size of one percent would indicate a . 546 percent increases in asked price. For bathrooms, an increase of $50 \%$, say from 2 to 3 , would indicate about an 18\% increase in asked price.

Both models provide useful information. The linear model could be used to assist a seller in pricing a house. The exponential model is useful for this, as well as estimating the value added to a house by increasing its size or number of bathrooms.
C. SALE PRICE AND SIZE, BEDROOMS, BATHROOMS AND TIME

This model examined the sale price as a function of size, number of bedrooms, number of bathrooms and quarter of sale. The prupose of this mo-el was to determine if there was a consistent relationship between sale price and the physical parameters and whether the time of the sale made a significant contribution to explaining the sale price.

## 1. Statistical Interpretation

For the linear model, the relationship estimated was

$$
X(2)=.445+1.197 X(3)+.783 X(5)
$$



The coefficients of $X(3)$ and $X(5)$ were significantly different from zero at the .01 level of significance. The coefficient of determination was .735, indicating a fair fit was obtained. The multicollinearity between $X(3)$ and $X(5)$ appeared in the same manner as in the previous model representing asked price. Neither the variable $X(4)$ nor the variables $X(7), X(8)$ and $X(9)$ passed a t-test that its coefficient was significantly different from zero at the .01 level of significance. As a result, they did not enter the model. The residuals from this model passed the test against heteroscedasticity. For the exponential model, the relationship estimated was

$$
X(2)=.854 x(3) .539 x(5) .374
$$

The exponents of $X(3)$ and $X(5)$ were both significantly different from zero at the . Ol level of significance. The coefficient of determination was .760 , indicating a slightly better fit than the linear model. The problem of multicollinearity between $X(3)$ and $X(5)$ showed itself as in the linear model. No other variables entered the model. The residuals from this model passed the test against heteroscedasticity. A comparison of the statistical attributes indicates that the exponential model is slightly superior.
2. Economic Interpretation

Both the linear and the exponential models indicated that, of the explanatory variables available, only the size of the house and the number of bathrooms contributed to
An $1+2-2+2-2+2$ $2+2$
正 P1 P1
explaining the behavior of the buyer and the seller in determining the sale price. This was similar to the results obtained in the Asked Price model. However, in this model the quarter of sale was available to assist in the explanation of the sale price. The fact that these variables did not enter the model caused rejection of the hypothesis that the time of year during which the sale made was significant.

With slight numerical differences, the economic interpretation given to the Asked Price model is also applicable to this model.
D. ASKED PRICE MINUS SALE PRICE AND ASKED PRICE AND TIME

This model examined the difference between asked price and sale price as a function of asked price and time. The purpose of the model was to determine if the difference between asked price and sale price increased as asked price increased and to examine whether this difference was influenced by the quarter of sale.

1. Statistical Interpretation

For the linear model the relationship estimated was

$$
X(6)=-0.080+0.070 X(1)
$$

The coefficient of $X(1)$ was significantly different from zero at the . 01 level of significance. The coefficient of determination was .203 , indicating a rather poor fit. As only one variable entered the model, there was no multicollinearity problem. The variables $X(7), X(8)$ and $X(9)$ did

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[^1]not pass a t-test that their coefficient was significantly different from zero. The residuals from this model passed the test against heteroscedasticity.

For the exponential model, the relationship estimated
was

$$
X(6)=-0.091 X(1)^{.218}
$$

The exponent of $X(1)$ was significantly different from zero at the . Ol level of significance. The coefficient of determination was . 136 , indicating a rather poor fit. Again, as only one variable entered the model, multicollinearity was not a problem. As with the linear model, the variables $\mathrm{X}(6)$, $X(7)$ and $X(8)$ did not pass a t-test for non-zero coefficients. The residuals from this model passed the test against heteroscedasticity.

## 2. Economic Interpretation

Neither of the models provide any positive economic insights. The primary inference drawn is negative. This is that, even though the magnitude of the asked explains very little of the variation in the difference between asked price and sale price, the quarter of sale does not explain enough of the remaining variation to allow it to enter the model as an explanatory variable. As with the previous attempt to use time as an explanatory variable, it did not have adequate contribution to make.


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## v. CONCLUSIONS

It has been shown that with a limited amount of data, it was possibly to develop useful models which describe the behavior of the buyers and sellers in a real estate market. Three of the four models explain over $70 \%$ of the variability in the dependent variable under consideration. The fourth, by its lack of ability to explain the variability in the dependent variable provides significant economic insights.

In summary, it was shown that the asked price was as excellent descriptor of the sale price. While this is not an unexpected result, the fact that there seems to be a consistent discount from the asked price is considered significant.

The second model illustrated that the size of the house was the most significant descriptor of asked price. The number of bathrooms was also of value in explaining some of the variation. In interpreting this model it must be remembered that the explanatory variables, in particular the number of bathrooms, may only be acting as proxies for other variables which were not available.

In examining the sale price as a function of the physical parameters and time, it was found that the time of sale was not an adequate explanatory variable. This finding is considered significant in that it is counter to the conventional wisdom of the real estate market. It disputes the ideas that summer is the best time to sell, sellers suffer a penalty if they must sell during the school year, etc.


The final model was valuable in confirming the findings of the third model with respect to the usefulness of time of sale as an explanatory varible. The difference between asked price and sale price by either asked price or tiem.

The results of this study are only applicable to the particular market examined at the particular time the data was recorded. Further, the models are only descriptive of the behavior of the participants in the market. They do not necessarily represent the actual considerations made by the buyers and the sellers.

2

## APPENDIX A

SUMMARY OF STATISTICAL RESULTS

SALE PRICE AND ASKED PRICE

LINEAR MODEL

| STEP | R2 | CONSTANT | X (1) |
| :---: | :---: | :---: | :---: |
| 1 | 0.978 | 0.080 | 0.930 |
| STD. ERR. |  |  | 0.010 |

CORRELATION MATRIX

|  | $\mathrm{X}(1)$ | $\mathrm{X}(2)$ |
| :--- | :--- | :--- |
| $\mathrm{X}(1)$ | 1.00 | 0.99 |
| $\mathrm{X}(2)$ | 0.99 | 1.00 |

HETEROSCEDASTICITY TEST - $\mathrm{F}=1.446$

EXPONENTIAL MODEL

| STEP | R2 | CONSTANT | X(1) |
| :---: | :---: | :---: | :---: |
| ${ }^{1}$ | 0.982 | -0.060 | 1.007 |
| STD. ERR. |  |  | 0.010 |

## CORRELATION MATRIX

|  | $\mathrm{X}(1)$ | $\mathrm{X}(2)$ |
| :--- | :--- | :--- |
| $\mathrm{X}(1)$ | 1.00 | 0.99 |
| $\mathrm{X}(2)$ | 0.99 | 1.00 |



## LINEAR MODEL

| STEP | R2 | CONSTANT | X(3) | X (5) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.663 | 0.815 | 1.975 |  |
| STD. ERR. |  |  | .103 |  |
| 2 | 0.734 | 0.433 | 1.271 | 0.834 |
| STD. ERR. |  |  | 0.137 | 0.120 |

CORRELATION MATRIX

|  | $\mathrm{X}(1)$ | $\mathrm{X}(3)$ | $\mathrm{X}(5)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{X}(1)$ | 1.00 |  |  |
| $\mathrm{X}(3)$ |  | 1.00 |  |
| $\mathrm{X}(5)$ |  |  | 1.00 |

HETEROSCEDASTICITY TEST - $\mathrm{F}=1.360$

EXPONENTIAL MODEL

| STEP | R2 | CONSTANT | X(3) | X(5) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.700 | 0.990 | 0.813 |  |
| STD. ERR. |  |  | 0.039 |  |
| 2 | 0.769 | 0.911 | 0.546 | 0.358 |
| STD. ERR. |  |  | 0.050 | 0.048 |

## CORRELATION MATRIX

|  | $\mathrm{X}(1)$ | $\mathrm{X}(3)$ | $\mathrm{X}(5)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{X}(1)$ | 1.00 | 0.84 | 0.79 |
| $\mathrm{X}(3)$ | 0.84 | 1.00 | 0.72 |
| $\mathrm{X}(5)$ | 0.79 | 0.72 | 1.00 |

HETEROSCEDASTICITY TEST - F = 1.174


## LINEAR MODEL

| STEP | R2 | CONSTANT | X(3) | X(5) |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{1}$ | 0.664 | 0.804 | 1.858 |  |
| STD. ERR. |  |  | 0.097 |  |
| $2^{2}$ | 0.735 | 0.445 | 1.197 | 0.783 |
| STD. ERR. |  |  | 0.128 | 0.112 |

## CORRELATION MATRIX

|  | $\mathrm{X}(2)$ | $\mathrm{X}(3)$ | $\mathrm{X}(5)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{X}(2)$ | 1.00 | 0.82 | 0.39 |
| $\mathrm{X}(3)$ | 0.82 | 1.00 | 0.74 |
| $\mathrm{X}(5)$ | 0.39 | 0.74 | 1.00 |

## HETEROSCEDASTICITY TEST - F = 1.244

EXPONENTIAL MODEL

| STEP | R2 | CONSTANT | X(3) | X(5) |
| :---: | :---: | :---: | :---: | :---: |
| I | 0.687 | 0.937 | 0.818 |  |
| STD. ERR. |  |  | 0.041 |  |
| 2 | 0.760 | 0.837 | 0.542 | 0.374 |
| STD. ERR. |  |  | 0.051 | 0.050 |

CORRELATION MATRIX

|  | $X(2)$ | $X(3)$ | $X(5)$ |
| :--- | :--- | :--- | :--- |
| $X(2)$ | 1.00 | 0.83 | 0.79 |
| $X(3)$ | 0.83 | 1.00 | 0.72 |
| $X(5)$ | 0.79 | 0.72 | 1.00 |

```
HETEROSCEDASTICITY TEST - F = 1.170
```

ASKED PRICE MINUS SALE PRICE AND ASKED PRICE AND TIME

## LINEAR MODEL

```
        STEP R2 CONSTANT X(1)
        llll
STD. ERR.
                                0.010
```

|  | $X(6)$ | $X(1)$ |
| :--- | :--- | :--- |
| $X(6)$ | 1.00 | 0.45 |
| $X(1)$ | 0.45 | 1.00 |

HETEROSCEDASTICITY TEST - F = 1.177

EXPONENTIAL MODEL

| STEP | R2 | CONSTANT | X (1) |
| :---: | :---: | :---: | :---: |
| 1 | 0.192 | -0.091 | 0.218 |
| STD. ERR. |  |  | 0.040 |

## CORRELATION MATRIX

|  | $X(6)$ | $X(1)$ |
| :--- | :--- | :--- |
| $X(6)$ | 1.00 | 0.37 |
| $X(1)$ | 0.37 | 1.00 |

HETEROSCEDASTICITY TEST - F = 1.096
-

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